Permutations

24 November 2020 09:46

SUM RULE

If r activities can be performed in n1, n2, ..., nr ways and if they are disjoint, viz., cannot be performed simultaneously, then any one of the r activities can be performed in (n1 + n2 + ... + nr) ways.

Q:→ In how many ways can we draw a

a) a heart or a spade

b) a numbered card or a king

c) a spade or an ace.

Sol a) A heart can be drawn in 13 ways and a spade can be drawn in 13 ways.

By the sum principle

Reqd. no. of ways to draw a heart or a spade $= 13+13$
$= 26$

b) A numbered card can be drawn in 36 ways and a king is drawn in 4 ways

By the sum principle

Reqd no. of ways to draw a numbered card or a king $= 36+4 = 40$.

c) A spade can be drawn in 13 ways and an ace can be drawn in 3 ways (As 1 ace is spade)

Reqd. no of ways $= 13+3 = 16$.

PERMUTATIONS WITH REPETITION

1) When repetition of n elements contained in a set is permitted in r-permutations, then the number of r-permutations is n^r.

Q:→ Consider 5 numbers 1, 2, 3, 4, 5. Find no. of

Q:→ Consider 5 numbers 1, 2, 3, 4, 5. Find no. of 3 digits number formed by these five numbers when repetition is allowed.

sol:→ Reqd. 3-digit numbers = $5 \times 5 \times 5 = 5^3 = 125$

2) The number of different permutations of n objects which include n1 identical objects of type I, n2 identical objects of type II, … and nk identical objects of type k is equal to

$$\frac{\lfloor n}{\lfloor n_1 \lfloor n_2 \cdots \lfloor n_k}$$

, where n1 + n2 + ….+ nk = n.

Q:→ Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements

i) do the words start with P.

ii) do all the vowels always occur together

iii) do the vowels never occur together

iv) do the words begin with I and end in P?

sol:→ INDEPENDENCE

No. of given letters = 12

No. of N's = 3

No. of D's = 2

No. of E's = 4

∴ Reqd no. of arrangements $= \frac{\lfloor 12}{\lfloor 3 \lfloor 2 \lfloor 4}$

$= \frac{\cancel{12} \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times \cancel{\lfloor 4}}{\cancel{6} \times \cancel{2} \times \cancel{\lfloor 4}}$

$= 1663200$

(i) Words starts with P

$\therefore$ Fix P at the beginning

Remaing 11 letters can be arranged in $\dfrac{\lfloor 11}{\lfloor 3 \lfloor 2 \lfloor 4}$

$$= \frac{11\times10\times9\times8\times7\times6\times5\times\cancel{\lfloor 4}}{6\times2\times\cancel{\lfloor 4}}$$

$$= 138600$$

(ii) Vowels occur together 7

I, EEEE

Five vowels as one letter

Also five vowels can be arrange themselves $\dfrac{\lfloor 5}{\lfloor 4}$ ways

We have 7 consonants and 1 letter assumption for the 5 vowels.

$\therefore$ we've to arrange 8 letters in $\dfrac{\lfloor 8}{\lfloor 3 \lfloor 2}$ ways

Reqd. No. of words in which vowels occur together

$$= \frac{\lfloor 8}{\lfloor 3 \lfloor 2} \times \frac{\lfloor 5}{\lfloor 4}$$

$$= \frac{8\times7\times6\times5\times4\times\cancel{\lfloor 3}}{\cancel{\lfloor 3}\times 2} \times \frac{5\times\cancel{\lfloor 4}}{\cancel{\lfloor 4}}$$

$$= 16800$$

(iii) Vowels never occur together

Reqd No. of words in which vowels never occur together = Total no. of words - Total no. of words in which vowels are together

$$= 16632400 - 16800 = 1646400$$

(iv) begin with I and end in P.
1 1

Reqd no of words $= \dfrac{\lfloor 10}{\lfloor 3 \lfloor 4 \lfloor 2}$

$= \dfrac{10 \times 9 \times 8 \times 7 \times \cancel{6} \times 5 \times \cancel{\lfloor 4}}{\cancel{6} \times \cancel{\lfloor 4} \times 2}$

$= 12600$

Q:→ How many numbers greater than 1000000 can be formed using the digits 1, 2, 0, 2, 4, 2, 4

sol:→ Tot. no. of digits = 7

No. of 2's = 3

" " 3's = 2

No. of digits to be taken at a time = 7

∴ No. formed $= \dfrac{\lfloor 7}{\lfloor 3 \lfloor 2} = 420$

No having 0 at the extreme left $= \dfrac{\lfloor 6}{\lfloor 3 \lfloor 2} = 60$

∴ Reqd. no. of numbers greater than 1000000

$= 420 - 60 = 360$